Multiple attenuation: Recent advances and the road ahead (2011)

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Multiple removal is a longstanding problem in exploration seismology. Although methods for removing multiples have advanced and have become more effective, the concomitant industry trend toward more complex exploration areas and difficult plays has often outpaced advances in multiple-attenuation technology. The topic of multiples, and developing ever more effective methods for their removal, remains high in terms of industry interest, priority and research investment. The question as to whether today, in 2011, multiples or multiple removal is winning is a way of describing what we are about to discuss. This paper focuses on recent advances, progress and strengths and limitations of current capability and a prioritized list of open issues that need to be addressed.

In seismic exploration it is useful to catalog events as primary- or multiple-based on whether the wave arriving at the receiver has experienced one or more upward reflection(s), respectively (Figure 1). Multiples are further subdivided and labeled according to the location of the downward reflection between two upward reflections. If the multiple has at least one downward reflection at the free surface, it is called a free-surface multiple, and if all of its downward reflections occur below the free surface, it is called an internal multiple. These definitions and cataloging of events into primary and multiple are operative and called upon only after the reference or background wavefield and the source and receiver ghosts have all been removed (Figure 2).

Both primaries and multiples contain information about the subsurface; however, (1) unraveling the information within a multiply reflected event is a daunting task, and (2) back-propagating a wavefield containing both primaries and multiples for imaging and inversion is usually beyond our ability to provide an accurate enough discontinuous overburden (required for migration and inversion). Hence, primaries are typically considered as signal and multiples are considered a form of coherent noise to be removed prior to extracting subsurface information from primaries.

"Multiple attenuation: an overview of recent advances and the road ahead" (Weglein, 1999) provides a 1999 perspective of multiple attenuation and places wave-theory advances at that time in the context of earlier pioneering contributions. We suggest *Multiple Attenuation* (published by SEG in 2005) and the special section on multiple attenuation (*TLE* 1999) as background to comprehend and to set the stage for this update and overview of recent progress, advances, and open issues as of 2011.

Offshore and onshore multiple removal: Responding to the challenges

In offshore exploration, the industry trend to explore in deep water, with even a flat horizontal water bottom and a 1D subsurface, immediately caused many traditional and useful signal processing/statistical-based multiple-removal methods to bump up against their assumptions, break down, and fail. In addition, marine exploration plays beneath complex multi-D laterally varying media and beneath and/or at corrugated, diffractive rapid varying boundaries (for example, subsalt, sub-basalt and subkarsted sediments and fault shadow zones) cause a breakdown of many other multiple-removal methods. For example, decon, stacking, *f-k,* Radon transform, and wavefield modeling and subtraction of multiples are among methods that run into problems with the violation of any one or a combination of the following assumptions: (1) primaries are random and multiples are periodic, (2) knowledge of the velocity of primaries and assuming the Earth has no lateral variation in properties with assumptions about 1D moveout, (3) velocity discrimination between primaries and multiples, (4) interpreter intervention capable of picking and discriminating primary or multiple events, and (5) determining the generators of the experiences of the multiples, and then modeling and subtracting them. The confluence of (1) high drilling costs in deepwater plays, (2) specific deepwater and shallow subsea hazards and technical challenges, (3) the need to develop fields with fewer wells, and (4) the record of drilling dry holes drives the need for greater capability for removing marine free-surface and internal multiples, as well as improving methods of imaging.

Moving onshore, the estimation and removal of land internal multiples can make the toughest marine-multiple problem pale in comparison. The presence of proximal and

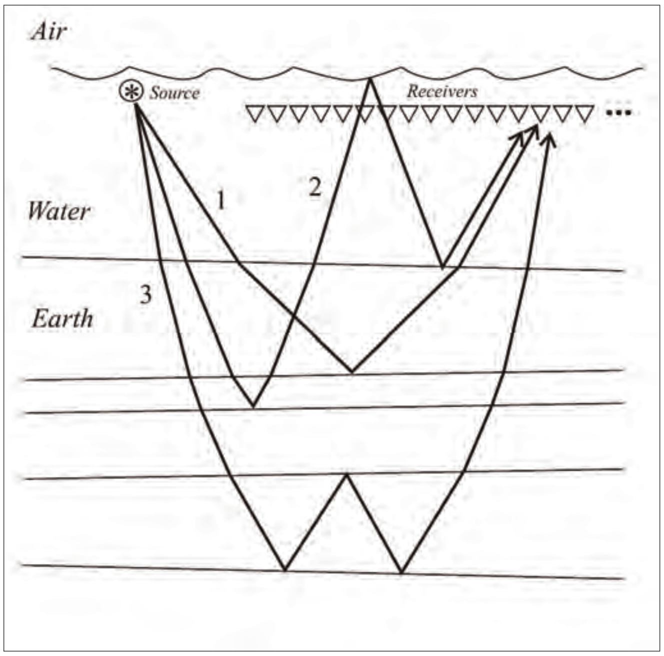


Figure 1. *Marine primaries and multiples: 1, 2 and 3 are examples of primaries, free-surface multiples, and internal multiples, respectively.*

interfering primaries and internal multiples of different orders can occur in marine situations, but their frequent occurrence for land internal multiples raises the bar of both the amplitude and phase fidelity of prediction and the priority and pressing need of developing an alternative to energy-minimizing-based adaptive subtraction techniques. For example, in Kelamis et al. (2006), Fu et al. (2010), and Luo et al. (in this special section), the basic cause of the land multiple-removal challenge in Saudi Arabia is identified as a series of complex, thin layers encountered in the near surface.

In general, strong reflectors at any depths can be identified as significant sources of internal multiples, especially where geologic bodies with different seismic properties are in contact. Typical examples are alternating sequences of sedimentary rocks and basaltic layers or coal seams, which can give rise to short-period internal multiples.

Multiples are a problem and a challenge due to violations of the assumptions and prerequisites behind methods used to remove them. There are two approaches to address those challenges: (1) remove the assumption violation (by satisfying the assumption), or (2) remove the assumption. That is, either develop a response and/or new methods that remove the violation, and arrange to satisfy the assumption, or develop fundamentally new methods that avoid the limiting or inhibiting assumption. There are cases and issues for which one or the other of these attitudes is called for and indicated. An example of seeking to satisfy a requisite is when a data acquisition is called for by a multiple-removal technique, and we seek methods of data collection and interpolation/extrapolation to remove the violation by satisfying the requirement. However, if a multiple-removal method is, for example, innately 1D in nature, then an interest in removing multiples in a multi-D Earth would call for developing a new method that did not assume a 1D Earth; i.e., it calls for developing a new multi-D method that altogether avoids the 1D assumption. The former, "remove assumption violation" approach would entail, e.g., arranging a 3D corrugated boundary subsalt play to somehow satisfy 1D layered Earth assumptions, velocity analysis, and moveout patterns, or modeling and subtraction of multiples, where seeking to satisfy those types of assumptions is not possible. The latter realization drove the search for new methods that avoid those increasingly difficult or impossible-to-satisfy criteria and prerequisites.

The list of sought-after characteristics for multiple attenuation

In response to those challenges, these new methods would therefore be required to satisfy the following criteria: (1) be fully multi-D, (2) make no assumptions about subsurface properties, (3) have no need for interpretive intervention, (4) be able to accommodate the broadest set of multiples of all orders, (5) extend to prime and composite events as introduced in Weglein and Dragoset (2005), where the definitions and meaning of primaries and multiples themselves can be extended from their original 1D Earth definitions and concepts, (6) be equally effective at all offsets, retaining effectiveness in prestack and poststack applications, and (7) last but not least, surgically remove multiples by predicting both their amplitudes and phases, and thus not harm primaries even if they are proximal and overlapping. The efficacy and choice among multiple-removal methods in response to the challenges posed in a world of complex multiple generators, in 1D Earth settings and/or in heterogeneous rapid laterally varying media and boundaries, would ultimately be evaluated, judged, and selected by how well they satisfy all of these criteria.

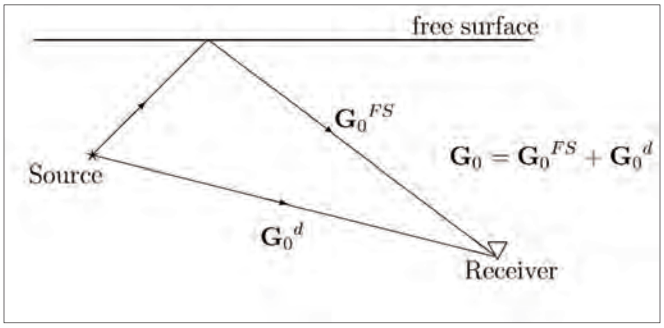


Figure 2. The marine configuration and reference Green's function.

The evolution and merging of methods that originally sought to either separate or wavefield-predict multiples

In Weglein (1999), multiple-removal methods were classified as: (1) separation and (2) wavefield prediction, and we refer the reader to Table 1 and Table 2 in that reference for a summary of methods within each category. Methods within the "separation" category were seeking a characteristic to separate primaries from multiples, whereas "wavefield prediction" was a way to wavefield-predict and then subtract multiples.

"Separation" methods were defined by characteristics that distinguish primaries from multiples, with, e.g., primaries considered as random and multiples as periodic, or assumptions about how primaries and multiples would separate in different transform domains. These methods earned their keep, but were ultimately hampered by their assumptions about the statistical nature of primary reflections, 1D Earth assumptions, and the assumed velocity determination for primaries.

"Wavefield-prediction" methods began with modeling and subtracting the entire history of the multiples that were targeted to be removed (e.g., Morley and Claerbout, 1983; Wiggins, 1988; Weglein and Dragoset, Chapter 4).

They moved away from 1D assumptions in principle, but were mainly confined to water-column reverberations, where they had demonstrated value, but had little hope or success in modeling and subtracting multiples with more complicated and sub-water-bottom experiences in their history.

The next step in "wavefield prediction" sought to not model the entire history of the multiple one wanted to remove, but rather to just find a wave-theory prediction to identify, isolate and separate the physical location and property that the multiple had experienced, and other events had not, and then to transform through a map of data with and without the experience as a way to "separate" events into those that have and have not had that experience. That thinking became the cornerstone of the "free-surface and interface method" pioneered and developed by Berkhout of the DELPHI Consortium at Delft University. That DELPHI program for removing all marine multiples required a sequence of relationships between data with and without isolated and well-defined reflections, starting with downward reflections at the air-water free surface, and then through a sequence of amplitude-preserving migrations, to image and transform away all internal multiples that had their shallowest downward reflection at each successively deeper reflector/interface starting at the water bottom. Hence, it's called the free-surface and interface method. That program provided significant added-value, especially with isolated free-surface multiples, or at times for internal multiples generated at a simple and not too complex water bottom. There was considerable reliance on "adaptive subtraction" to fix omissions in the theory and limitations in data collection and prerequisites like deghosting and wavelet removal. The DELPHI approach is a wavefield-prediction method that doesn't require modeling the entire history and experience of the multiple, as earlier wavefield-prediction methods required, but required only modeling in detail the "wavefield-prediction" properties that "separated" the events experiencing a shallowest downward reflection at the free surface, and then repeating that program at the next interface or boundary in a sequence of deeper interfaces. Events are thus separated by whether they have or have not had a downward reflection at those reflecting boundaries. Hence, "wavefield prediction" and "separation" merged, with the separation requiring detail of all subsurface properties down to and including a given interface to remove all multiples having a shallowest reflection at that interface. However, that comprehensive program ran into problems of conceptual and practical issues, with the former, including: (1) how to transform away via, e.g., Green's theorem a relationship between data experiencing and not experiencing a corrugated and diffractive boundary, and, (2) the stringent requirements of determining the properties above, and down to, and at, the interface. The latter issues made the use of these interface internal multiple-removal methods difficult to be applied in practice as targets became deeper and the overburden and interfaces became rapidly varying and difficult to adequately identify.

The inverse scattering series (ISS) methods for removing free-surface and internal multiples can be viewed as representing the next step in the evolution of "separation" and "wavefield-prediction" concepts and methodology. The ISS methods are in some sense a direct response to the limitations of the DELPHI free-surface and interface approach, with (1) a more complete free-surface removal, in terms of amplitude and phase at all offsets, and (2) an internal multiple-removal method that did not require any subsurface information whatsoever. There are "wavefield-prediction" and "separation" ingredients in the ISS free-surface and internal multiple-removal methods. For free-surface multiple removal, the free-surface properties are assumed to be known, and a subseries of the inverse scattering series "separates" deghosted data with free-surface multiples from deghosted data without

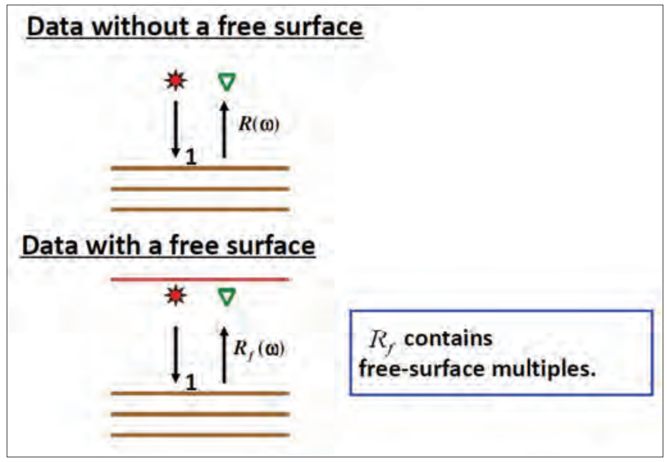
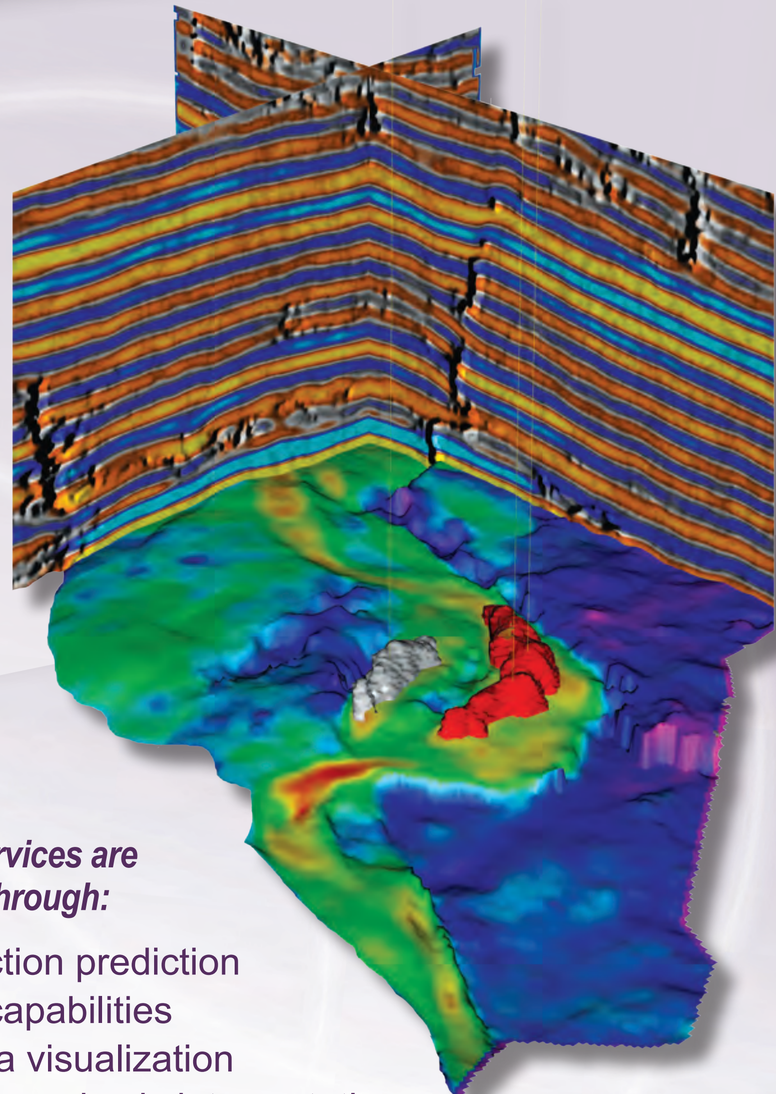


Figure 3. *Data without a free surface (top) and with a free surface (bottom).*

free-surface multiples. The ISS free-surface multiple separation is realized by the actual location and physical properties that free-surface multiples have experienced at the free surface, distinguishing themselves from data/events that have not shared that free-surface experience. For internal multiples the inverse scattering series takes on another attitude. The forward series allows the construction of primaries and internal multiples through a description entirely in terms of water speed and, through the reverse, the seismic processing or inverse scattering series, in turn, allows for the removal of internal multiples, and the depth imaging and inversion of primaries directly in terms of water speed. For internal multiple removal there is no downward continuation into the Earth, no interface identification and removal. The "separation" between primaries and internal multiples in the forward or data creation scattering series and inverse or data processing, inverse scattering series, is carried out by understanding how primaries and internal multiples differ in their forward construction, in terms of a water speed picture/construction, and then how to separate the removal of internal multiples from the imaging and inversion of primaries, also directly and only in terms of data and water speed. In contrast to the DELPHI internal multiple interface method, the ISS internal multiple-removal method never requires, determines or estimates the actual subsurface medium properties and interfaces the internal multiple experiences. The inverse scattering series multiple-removal methods are flexible, allowing (1) the separation to be in terms of distinguishing by whether or not the event has a certain well-located and well-defined experience in its history, where the actual medium properties are available and reliable, as occurs with the free surface and in ISS free-surface multiple-removal algorithm, and (2) without knowing or needing to determine anything about the actual separating experience for ISS internal multiple removal. The ISS separation of the imaging and inversion of primaries from the removal of internal multiples thus avoids all of the conceptual and practical limitations of the DELPHI free-surface and interface approach, and ultimately accounts for its current position as stand-alone for addressing the most difficult



and daunting marine and land internal multiple challenges.

The two classic multiple-r emoval categories separation, and wavefield prediction, have evolved and merged into the maximally flexible, accommodating and effective inverse scattering series multiple-removal methods: prediction and separation of events either with or without needing, kno wing or determining the location and physical properties of the experience (e.g., a free surface or subsurface reflector, respectively) that separates ev ents into two categories—ev ents that hav e, and events that have not, experienced in their histor y a shallowest downward reflection at a specifi c reflector, and without the need for any subsurface information, event picking or interpreter intervention. The ISS allows all internal multiples to be predicted and separated from all reflectors, at all depths, at once, without knowing, needing, or determining anything about those reflectors. The inverse scattering series multiple-removal methods hav e incorporated the str engths of earlier separation and wav efield-prediction concepts and thinking, while avoiding the practical limitations, drawbacks and weaknesses of earlier and competing approaches.

Before discussing, classifying, and comparing methods for removing multiples, it will be useful to intr oduce and briefly discuss two impor tant background topics/subjects that will enhance and facilitate understanding the sometimes counter-intuitive ideas we will be describing and attempting to convey.

Modeling and inversion are two entirely different enterprises

In this paper, we adopt an inclusive definition of inversion that includes any method that determines subsurface properties from measured surface data, or any intermediate task (e.g. multiple removal or depth imaging) toward that goal. Inversion methods can be direct or indirect, and these approaches are not in any practical or theoretical sense the same or equivalent. Modeling run backward, or model matching or iterative linear inverse model matching, or any form of indirect inversion, or solving a direct forward problem in an inverse sense, are not equivalent to d irect inversion. Nor is any intermediate seismic processing objective, within a direct inversion algorithm, equivalent to solving for that same goal in some model-matching or indirect manner. That statement is true independent of: (1) the capability and speed of your computer, (2) the nature of the objective function, and (3) the local or global search engine. The only exception to that rule is when the direct inverse task is linear (e.g., when the goal is depth imaging and *you know the velocity field*, the direct inverse for depth migration is linear, and then modeling run backward is direct depth imaging). If the direct inverse is nonlinear in either the entire data set or a si ngle event, then modeling run backward is not the equivalent of a direct inverse solution. There is widespread confusion on this fundamental and central point within math, physics, and geophysics "inversion" circles with significant and harmful conceptual and practical real-world consequence. See Weglein et al. (2009) for full detail and examples. And it is worth noting at this point that the inverse scattering series is the only direct inverse for a m ultidimensional acoustic, elastic, or inelastic heterogeneous Earth.

Prediction and subtraction: The plan to strengthen the prediction, and reduce the burden, dependence and mischief of the subtraction

Multiple removal is often described as a two-step procedure: prediction and subtraction. The subtraction step is meant to try to compensate for any algorithmic compromises, or real world conditions, outside the physical framework behind the prediction. In multiple-removal applications, the subtraction step frequently takes the form of energy-minimizing adaptive subtraction. The idea is that a section of data (or some temporally local portion of data) without multiples has less energy than the data with multiples. One often hears that the problem with multiple attenuation is not the prediction but the subtraction. In fact, the real problem is excessive reliance on the adaptive subtraction to solve too many problems, with an energy-minimizing criteria that can be invalid or fail with proximal or overlapping events. The breakdown of the energy-minimization adaptive subtraction criteria itself can occur precisely when the underlying physics behind, e.g., high-end inverse scattering series multiple prediction (that it is intended to serve) will have its greatest strength and will undermine rather than enhance the prediction.

The essence of ISS: An important prototype example

We will demonstrate some of these ideas (using a 1D plane-wave normal incidence case) for the inverse scattering free-surface multiple elimination method. There are other ways to derive the free-surface multiple-removal algorithm (e.g. Ware and Aki, 1968; Fokkema and van den B erg, 1990), but the ISS is unique in its message that all processing goals (e.g., internal multiple removal, depth imaging, nonlinear direct target identification, and Q-compensation without Q) can each be achieved in the same manner that the ISS removes free-surface multiples, i.e., directly without subsurface information. Hence, this analysis below carries much broader consequences beyond the immediate goal of the ISS removing free-surface multiples.

Figure 3 describes a situation in which a unit-amplitude downgoing wave leaves a sour ce in the water column. The upper figure assumes that ther e is no fr ee surface. $R(\omega)$ denotes the single temporal fr equency of the upgoing recorded field. The lower figure corresponds to the same situation with the addition of the fr ee surface. $R_f(\omega)$ is the single tempo-

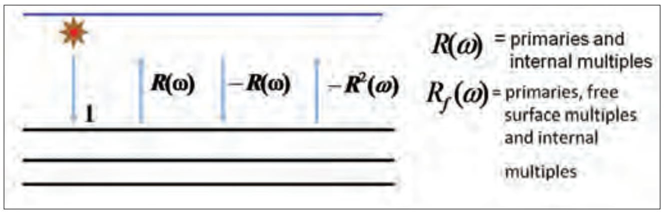


Figure 4. *The forward problem. Constructing free-surface multiples [i.e., from* R(ω) *to* $R_f(\omega)$*)].*

ral frequency of the upgoing portion of the recorded data. $R(\omega)$ contains all primaries and internal multiples. $R_f(\omega)$, on the other hand, is the upgoing portion of the total measured wavefield and consists of primaries, internal multiples, and free-surface multiples. The downgoing source wavefield and the upgoing receiver wavefield would be realized in practice by source and receiver deghosting. Source and receiver deghosting is a critically important step to assure subsequent amplitude and phase fidelity of the ISS free-surface multiple-removal methods, whose derivation follows below.

Forward construction of data with free-surface multiples, $R_f(\omega)$ in terms of data without free-surface multiples, $R(\omega)$

The downgoing source wavefield of unit amplitude first impinges on the Earth and $R(\omega)$ emerges (consisting of all primaries and internal multiples). $R(\omega)$ hits the free surface and $-R(\omega)$ is the resulting downgoing wave (because the reflection coefficient is -1 for the pressure field at the free surface). This downgoing field, $-R(\omega)$, in turn enters the Earth as a "wavelet", and $-R^2(\omega)$ emerges, and this repeats in the manner shown in Figure 4.

The total upgoing wavefield in the presence of a free surface, $R_f(\omega)$, is expressed in terms of the total upgoing wavefield in the absence of the free surface, $R(\omega)$:

$$R_f(\omega) = R(\omega) - R^2(\omega) + R^3(\omega) + \cdots \quad (1)$$

$$R_f(\omega) = R(\omega)/[1 + R(\omega)] \quad (2)$$

Several points are worth noting about this result.

The inverse series for removing free-surface multiples corresponding to the forward series (Equation 1) that constructs free-surface multiples is found by rearranging Equation 2 into $R = R_f/(1-R_f)$ and then expressing R as the infinite series

$$R = R_f + R_f^2 + R_f^3 + \cdots \quad (3)$$

This expression is, indeed, the 1D normal-incidence version of the inverse scattering free-surface multiple-attenuation algorithm (Carvalho, 1992; Weglein et al., 1997). Notice that neither the forward (construction) series for R_f in terms of R nor the removal (elimination) series for R in terms of R_f depend on knowing anything about the medium below the receivers.

The ISS free-surface removal series derivation and algorithm (Equation 3) does not care about the Earth model type and is completely unchanged if the Earth is considered to be acoustic, elastic, or anelastic. That property is called "model type independence," (see Weglein et al., 2003).

The derivation of these series (Equations 1 and 3) was based on the difference in the physical circumstances that gives rise to the events we are trying to isolate and separate: free-surface multiples and the (−1) reflection coefficient at the free surface (the physical circumstance).

Both the construction and elimination process assume a wavelet deconvolution in the forward problem. The wavelet, $S(\omega)$, plays a role in the forward problem:

$$R_f(\omega) = S(\omega)R(\omega)/[1 + R(\omega)],$$

and in the inverse

$$R = (R_f/S)/(1-(R_f/S)) = R_f/S + R_f^2/S^2 + R_f^3/S^3 + \cdots.$$

where the meaning of the quantity R_f is $S(\omega)$ times R_f in Equations 1 and 2. Hence, for free-surface multiple removal, there is a critical need for the wavelet because the effectiveness of the series has a nonlinear dependence on $1/S(\omega)$.

Free-surface demultiple algorithm: Instructive analytic examples

We present an analytic 1D normal incidence example (Figure 5) to illustrate the inner workings of the ISS free-surface multiple-removal algorithm.

The reflection data in the time domain are expressed as

$$R_f(t) = R_1\delta(t-t_1) + R_2'\delta(t-t_2) - R_1^2\delta(t-2t_1) - R_2'^2\delta(t-2t_2) - 2R_1R_2'\delta(t-(t_1+t_2)) + \cdots,$$

where R_1 and R_2' are the amplitudes of the two primaries in this two reflector example. In the frequency domain,

$$R_f(\omega) = R_1e^{i\omega t_1} + R_2'e^{i\omega t_2} - R_1^2e^{2i\omega t_1} - R_2'^2e^{2i\omega t_2} - 2R_1R_2'e^{i\omega(t_1+t_2)} + \cdots,$$

and

$$R_f^2(\omega) = R_1^2e^{2i\omega t_1} + R_2'^2e^{2i\omega t_1} + 2R_1R_2'e^{i\omega(t_1+t_2)} + \cdots.$$

Hence $R_f(\omega) + R_f^2(\omega)$ precisely eliminates all free-surface multiples that have experienced one downward reflection at the free surface. The absence of low frequency (and in fact all other frequencies) plays absolutely no role in this prediction. This is a nonlinear direct inverse that removes free-surface multiples. There is no imaginable way that one frequency of data could be used to model and subtract one frequency of free-surface multiples. A single frequency of data cannot even locate the water bottom. This is an example of how a direct nonlinear inverse does not correspond to a forward problem run backward. Furthermore, model matching and subtracting multiples are inconceivable without knowing or caring about the Earth model type for the modeling step. This illustrates how model matching, iteratively or otherwise, modeling run backward, and all forms of indirect inversion are not equivalent to a direct inverse solution.

Recovering an invisible primary

Consider a free-surface example (Figure 6) with the following data, corresponding to two primaries and a free-surface multiple:

$$R_t(t) = R_1\delta(t-t_1) + R_2'\delta(t-t_2) - R_1^2\delta(t-2t_1). \quad (4)$$

Now assume for our example that

$$R_2' = R_1^2,$$

$$t_2 = 2t_1.$$

Then from Equation 4,

$$R_f(t) = R_1\delta(t - t_1).$$

The second primary and the free-surface multiple cancel, and

$$R_f(\omega) = R_1 e^{i\omega t_1}$$

$$R_f^2(\omega) = R_1^2 e^{2i\omega t_1}$$

$$R_f(\omega) + R_f^2(\omega) = R_1 e^{i\omega t_1} + R_1^2 e^{2i\omega t_1}$$

$$R(t) = R_1\delta(t-t_1) + R_1^2\delta(t-2t_1) = R_1\delta(t-t_1) + R_2'\delta(t-2t_1)$$

resulting in the two primaries by recovering the primary not "seen" in the original data.

The ISS free-surface multiple-removal algorithm, with deghosted and wavelet deconvolved data, can predict and subtract the hidden multiple and recover the hidden primary. If these obliquity factor deghosting and wavelet ingredients are compromised in the prediction, the amplitude and phase will be incorrect and the invisible primary will not be recovered. Furthermore, when the multiple is removed in the invisible reflector example, the energy goes up, not down, and the adaptive subtraction energy-minimization criterion fails and cannot "fix" the problem caused by missing obliquity factors, wavelet removal, and deghosting. The lesson: Don't compromise on prediction strengths and assume the subtraction (adaptive) will atone for any shortcomings. The ISS FS multiple prediction has no trouble recovering the hidden primary. Zhang (2007) demonstrates with a prestack example that with deghosted data the ISS free-surface algorithm precisely predicts the FS multiple without the need for adaptive subtraction. For these same examples and in general, the feedback loop free-surface multiple-attenuation algorithm, with its lack of an obliquity factor and retaining the source-side ghost, will not accurately predict the amplitude and phase of free-surface multiples.

ISS internal multiple-attenuation algorithm

The ISS internal-multiple-attenuation algorithm in 2D starts with the input data, $D(k_g, k_s, \omega)$, that is deghosted, wavelet deconvolved, and with free-surface multiples removed. The parameters, k_g, k_s, and ω, represent the Fourier conjugates to receiver, source, and time, respectively. The ISS internal-multiple-attenuation algorithm for first-order internal multiple prediction in a 2D Earth is (Araújo, 1994; Weglein et al., 1997).

$$b_{3IM}(k_g, k_s, \omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_2(z_g - z_s)} \int_{-\infty}^{\infty} dz_1 b_1(k_g, k_1, z_1) e^{i(q_g+q_1)z_1} \times \int_{-\infty}^{\infty} dk_2 e^{-iq_1(z_g - z_s)} \int_{-\infty}^{z_1-\varepsilon} dz_2 b_1(k_1, k_2, z_2) e^{-i(q_1+q_2)z_2} \int_{z_2+\varepsilon}^{\infty} dz_3 b_1(k_2, k_s, z_3) e^{i(q_2+q_s)z_3} \quad (5)$$

In the previous Equation 5, the quantity $b_1(k_g, k_s, z)$ corresponds to an uncollapsed migration (Weglein et al., 1997) of an effective incident plane-wave data. The vertical wavenumbers for receiver and source, q_g and q_s are given by $q_i = \mathrm{sgn}(\omega)\sqrt{(\omega/c_0)^2 - k_i^2}$ for $i = (g,s)$; c_0 is the constant reference velocity; z_s and z_g are source and receiver depths; and z_i (i = 1, ... ,3) represents pseudodepth. $b_{3IM}(k_g, k_s, \omega)$ is a portion of a term in the ISS that performs prediction of all first-order internal multiples at all depths at once.

For a 1D Earth and a normal-incidence plane wave, Equation 5 reduces to

$$b_{3IM}(k) = \int_{-\infty}^{\infty} dz_1 b_1(z_1) e^{ikz_1} \int_{-\infty}^{z_1-\varepsilon} dz_2 b_1(z_2) e^{-ikz_2} \int_{z_2+\varepsilon}^{\infty} dz_3 b_1(z_3) e^{ikz_3} \quad (6)$$

For the example shown in Figure 6 with two primaries:

$$b_1(t) = R_1\delta(t - t_1) + R_2'\delta(t - 2t_2)$$

We transform the data into pseudodepth:

$$b_1(z) = R_1\delta(z - z_1) + R_2'\delta(z - 2z_2)$$

where $z_1 = \frac{c_0 t_1}{2}$ and $z_2 = \frac{c_0 t_2}{2}$. The integral in Equation 6 produces

$$b_{3IM}(k) = e^{2ikz_2 - ikz_1} R_1 R_2^2 T_{01}^2 T_{10}^2,$$

and in the time domain:

$$b_{3IM}(t) = \delta(t - (2t_2 - t_1)) R_1 R_2^2 T_{01}^2 T_{10}^2.$$

The actual internal multiple is

$$-\delta(t - (2t_2 - t_1)) R_1 R_2^2 T_{01} T_{10}.$$

Hence, Equations 5 and 6 predict the precise time and approximate amplitude of the internal multiple (i.e., it's an attenuator). There is a closed form subseries of the ISS that eliminates that multiple (Ramirez and Weglein, 2005).

Examples of 2D ISS free-surface and internal multiple removal with marine data

Figure 7 shows an example of the inverse scattering series internal-multiple-attenuation algorithm applied to a 2D synthetic data set. The data were computed using an Earth model characterized by rapid lateral variations (Figure 7a). In Figure 7, from left to right, the three panels show the input data, the predicted internal multiples, and the result of inverse scattering internal multiple attenuation, respectively.

Figures 8a and 8b illustrate the free-surface and internal multiple-attenuation algorithms applied to a data set from the Gulf of Mexico over a complex salt body. Seismic imaging beneath salt is a challenging problem due to the complexity of the resultant wavefield. In Figure 8a, the left panel is a

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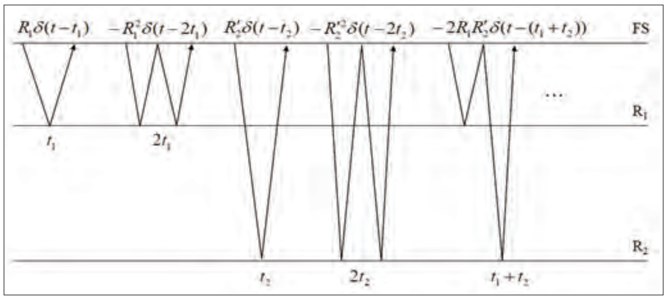

Figure 5. An analytic 1D normal incidence example to illustrate the inner workings of the ISS free-surface multiple-removal algorithm.

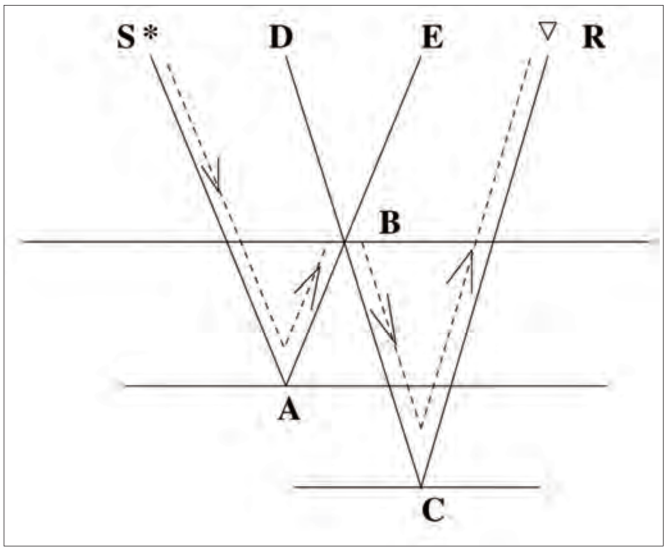

Figure 6. A one-dimensional model with two interfaces.

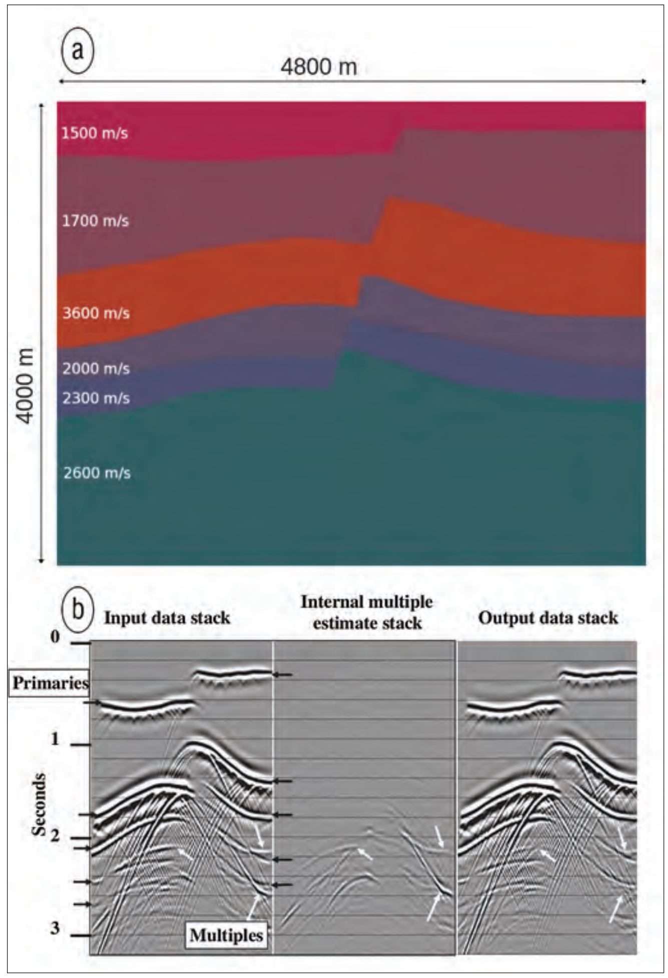

Figure 7. (a) A 2D synthetic model characterized by gently curved reflectors intersected by a fault. (b) The left panel shows a common-offset display from the synthetic data set created using the model. The middle panel shows the predicted internal multiples and the right panel is the result after subtracting the predicted multiples from the input data set. (From Matson et al., 1999, and Weglein et al., 2003)

stacked section of the input data and the right panel shows the result of the inverse scattering free-surface multiple-removal algorithm. Figure 8b illustrates the internal-multiple-attenuation method applied to the same Gulf of Mexico data set. An internal multiple that has reverberated between the top of the salt body and the water bottom (and interferes with the base salt primary) is well attenuated through this method.

ISS internal multiple application for land

Fu et al. (2010), along with Terenghi et al. and Luo et al., (in this special section) describe the motivation, evaluation, and comparison of different approaches to removing internal multiples on complex synthetic and onshore data. Fu et al. concluded that "Their (ISS internal multiple algorithm) performance was demonstrated with complex synthetic and challenging land field data sets with encouraging results, where other internal multiple suppression methods were unable to demonstrate similar effectiveness."

While the ISS internal multiple attenuator was unmatched in capability, in comparison with other internal multiple methods tested, an examination of the results shows that there are open issues yet to be addressed. A more complete understanding of the action of the ISS first-order internal multiple attenuator (Equation 5) when the input consists of all the events in the recorded data, and the anticipated need for further inclusion of ISS internal multiple-removal capability in our algorithm are our response to those issues, and are currently underway.

The Delft group, led by Berkhout, at some point several years ago took note and acknowledged the ISS internal multiple approach and then formulated several new and innovative DELPHI approaches that drew upon certain (but not all) aspects and properties of the ISS internal multiple algorithm. The differences between the latter DELPHI approaches and the ISS internal multiple method today remain significant and substantive. The comparisons to ISS internal multiple attenuation referred to in Fu et al. included the DELPHI approaches to internal multiple removal. The details behind the Fu et al. tests and results are described, explicated and further analyzed in Terenghi et al.

Discussion

We have described a "wish list" of qualities that the ideal response to multiple-removal challenges would satisfy, and

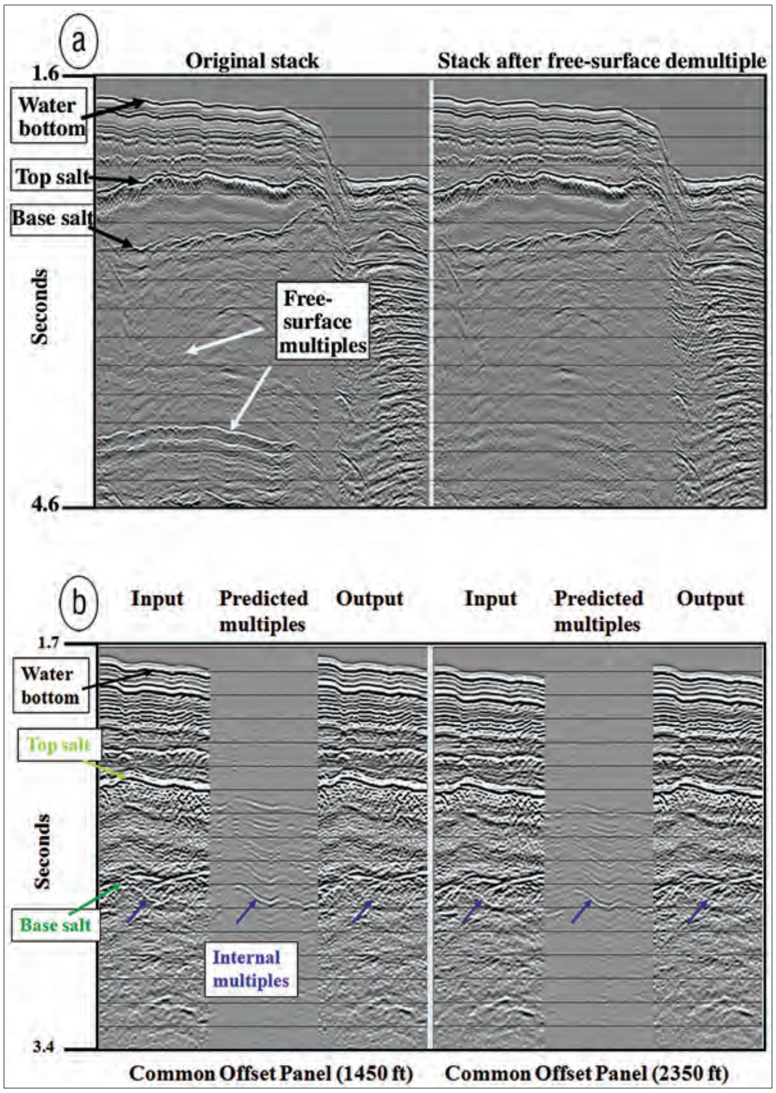

Figure 8. *(a) The left panel is a stack of a field data set from the Gulf of Mexico. The right panel is the result of ISS free-surface multiple removal. (b) The ISS internal multiple-attenuation method applied to the same data set after free-surface multiple removal. Data courtesy of WesternGeco. (From Matson et al., 1999, and Weglein et al., 2003)*

have shown that only the ISS multiple-removal methods are candidates toward reaching that high standard. All methods have strengths and shortcomings, and as we recognize the shortcomings of the current ISS attenuator, we also recognize that removing them resides within the ISS and that "upgrade" will never require subsurface information, picking events or any interpretive intervention or layer stripping. What all the ISS methods require is a reasonable source signature and deghosting, and we are developing onshore Green's theorem methods for that purpose (see Zhang and Weglein, 2005; Zhang and Weglein, 2006; and Mayhan et al., 2011).

Adaptive energy-minimizing criteria are often employed in an attempt to bridge the conditions and limitations of the real world and the physics behind what our algorithms are assuming. When first introduced by Verschuur et al. (1992) and Carvalho and Weglein (1994), the need was clear and good benefit was derived, especially with isolated primaries and free-surface multiples of first-order. But, as with all assumptions, today's reasonable and necessary assumption will invariably be tomorrow's impediment to progress and increased effectiveness. And that's the case with adaptive subtraction today, especially with land and complex marine internal multiples. We have advocated a three-pronged response to land and complex marine internal multiples: (1) seeking further capability for amplitude fidelity for all orders of internal multiples, including converted-wave internal multiples, (2) satisfying prerequisites for the source signature and radiation pattern, and (3) look for a new "bridge" to replace the energy-minimization adaptive criteria, a bridge consistent with the underlying physics rather than running at cross purposes with the greatest strength of the ISS prediction. For marine multiple removal, a key impediment for shallower-water exploration is the inability to extrapolate to near-source precritical angle traces when the nearest receiver is in the postcritical region. That can shut down free-surface multiple removal and can impede interpretation and drilling decisions. All methods for extrapolation—including *f-k*, Radon, interferometry (i.e., Green's theorem), and migrate demigrate data reconstruction—fail to provide that post- to precritical curve-jumping capability. One possibility with some ray of hope and optimism is to invert the postcritical data with model matching (Sen et al., 2001). That global search procedure and test, although positive and encouraging, was already pushing compute and algorithm capability with an initial 1D elastic test and application. Further attention and progress on this open issue is warranted and could pay significant dividends. Our plan is to progress each of these issues as a strategy to extend the current encouraging results and allow ISS multiple removal to reach its potential: to surgically remove all multiples without damaging primaries under simple or complex, daunting land and marine circumstances.

Summary

The strategy that we advocate is a tool-box approach, where the appropriate multiple-removal method is chosen, based on the given data set and the processing goal. The relative use of different methods within the tool box has shifted over time as exploration portfolios have focused on more remote, complex and difficult marine and land plays. That industry trend and need drives our orientation and continued interest in multiple removal. Its objectives are: (1) fidelity of both amplitude and phase prediction to allow surgical multiple removal of all multiples without damaging primaries; (2) including all relevant multiples in the algorithms; (3) using appropriate orders of multiple-removal terms from ISS multiple-removal subseries) in the prediction; (4) strengthen the prediction and reduce the burden on the adaptive subtraction, and (5) develop a replacement to the energy-minimization criteria that will align with rather than impede the method it is meant to serve. The ISS methods for removing free-surface and internal multiples are an essential and uniquely qualified ingredient/component in this strategy. When other priorities (like cost) might reasonably override the interest in (1) amplitude and phase fidelity, (2) inclusion of all internal multiples, and/or when the generators of the relevant internal multiples can be reliably identified, then the DELPHI methods can be the appropriate and indicated choice.

The potential cost of drilling dry holes always has to be taken into account. The industry move to 3D acquisition and processing was not put forth to save money on acquisition and processing—it saved money by drilling fewer expensive dry holes. One exploratory well in the deep water Gulf of Mexico can cost US $200 million—and we can significantly increase data acquisition investment and processing expenditure by the cost saving of avoiding dry holes and improving the exploration drilling success rate. Distinguishing between a multiple and a gas sand is a "drill no-drill" decision.

In summary, multiple-removal prediction methods have progressed and there is much to celebrate. The capability and potential that resides within the ISS for attenuating multiples has already shown differential added value. However, the trend to more complex and challenging marine and onshore plays demands inclusiveness of all troublesome multiples in the removal, along with: (1) stronger and more competent prediction, with amplitude and phase fidelity at all offsets, and (2) the development of fundamentally new concepts and criteria for subtraction, that align with rather than undermine the strengths of high-end prediction. There will always be a need for a subtraction step, attempting to deal with issues beyond the framework of the prediction, and there will always be those types of "beyond the framework" issues. We need a more sophisticated and capable subtraction criteria. The adaptive subtraction concept has been enormously useful, with a strong record of contribution but it is now too blunt an instrument for the more complicated and complex challenges. In the interim, the strategy is to build the strength of the prediction and to reduce the burden on the adaptive subtraction. The ISS is also the source of an effective response to outstanding open issues on amplitude and all orders of internal multiples which have moved from the back burner to center stage. The key to that strategy builds predictive strength from a direct inverse machinery, and wave-theory-deterministic Green's theorem prerequisite satisfaction, while seeking near-term reduction of the burden on the energy-minimization adaptive subtraction, and ultimately to replace the latter with an entirely consistent, comprehensive and more effective prediction and subtraction of multiples. The ISS multiple prediction, and the Green's theorem prerequisite satisfaction for the data wavelet and deghosting, are aligned and consistent. A subtraction on that same footing would provide an overall comprehensive and consistent methodology and a step improvement in multiple-removal capability. In this paper, we want to communicate our support and encouragement for that necessary future development and delivery.

The progress and success represented by advances in multiple-attenuation methods has given hope to heretofore areas that were previously "off-limits" and "no-go zones." That, in turn, has allowed our industry to imagine that yet more difficult exploration areas and targets could be accessible. In summary, that is the encouraging and positive response to the question "multiples or multiple removal; who is winning?"

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